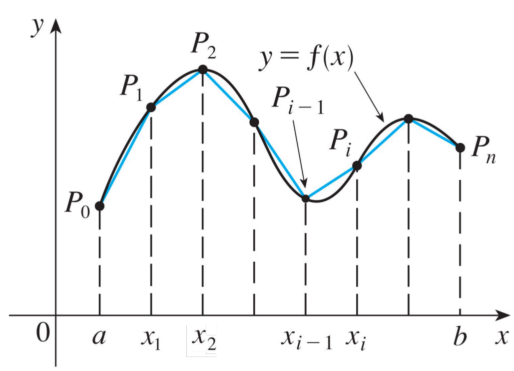
Group Project 5
Arc Length
Due Friday, Mar. 18

*****Read this first!** – You will need to email your professor (me) to get the specific curve to use in your report. Be sure to do this early enough to get this data and complete your report. (e,g. if you email me at 11:00 PM, I probably will not get you the data until the following day.)

Project Assignment: Imagine a situation where you have some curve and you need to know its length. Suppose want to build something and you need to know the length of some part before you start. How can you measure the length without building the actual object first? Even if it were built, how can you measure the length of a curve? A ruler won't work and a tape measure might be a complete pain!

In this project you will develop a method to calculate the arc length of a curve. Suppose you have a curve C defined by a continuous function $y = f(x)$ for $a \leq x \leq b$. The way we will calculate the arc length, L, of the curve is very similar to the idea of integration in general. Be will break the interval $[a, b]$ into n subintervals with endpoints $a = x_0, x_1, x_2, ..., x_n = b$ and equal width Δx.

We will approximate the curve with straight lines on each subinterval. If P_i is the point $(x_i, f(x_i))$ for each $0 \leq i \leq n$, then we approximate the arc of $f(x)$ between P_{i-1} and P_i by a straight line segment connecting these two points.



The length of the line segment connecting P_{i-1} and P_i is written $|P_{i-1}P_i|$. We can approximate the arc length of the entire curve by summing the lengths of each of the subintervals.

$$L \approx \sum_{i=1}^{n} |P_{i-1}P_i|$$

1. Using the Pythagorean theorem and $n = 4$ subintervals, approximate the arc length of the curve assigned to you by your professor.

2. Repeat this process with $n = 8$ subintervals.

3. To get the actual arc length, L, of the curve we need to take the limit of the sum as n goes to infinity. To do this, write $\Delta x = x_i - x_{i-1}$ for $0 \leq i \leq n$ and $\Delta y_i = y_i - y_{i-1}$. Write the length of line segment i in terms of Δx and Δy_i.

4. Rewrite the length of line segment i by pulling out a Δx. Write all remaining Δx and Δy_i terms in the form
$$\frac{\Delta y_i}{\Delta x}.$$

5. Now calculate the limit at n goes to infinity.
$$l = \lim_{n\to\infty} \sum_{i=1}^{n} |P_{i-1}P_i|$$
Recall that $n \to \infty$ is the same as $\Delta x \to 0$, that $\displaystyle\lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x} = f'(x)$, and that $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} f(x)\Delta x = \int_a^b f(x)\,dx$.

6. Use these results to calculate the exact arc length of the curve assigned to you by your professor. You may want to use a computer to help evaluate this integral. Some calculators can do this and WolframAlpha.com is a good resource.

Your group should print your report and turn it in by the due date. Please be sure to fill out and return a project assessment form with your report.